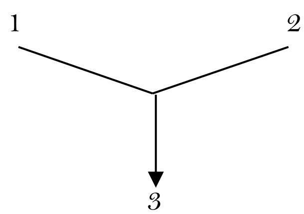
Reductio ad absurdum

Reading (before seminar)

Here you will learn a form of argument known as *reductio ad absurdum* – "reduction to absurdity," also known simply as a *reductio*. Such an argument is an attempt to show that some claim x is false by demonstrating that an absurd conclusion follows from the supposition that x is true. Here we will not examine all forms of argument by *reductio*, but only the simple case where the absurd conclusion is a contradiction. Any such argument has the following form:

1. If x, then y.
2. If x, then not-y.
3. Not-x.



Consider, for example, the following well-known mathematical *reductio*[1]:

> Suppose that there are only finitely many prime numbers, $p_1 \ldots p_n$. Then consider the number $p = p_1 \times p_2 \times \ldots \times p_n + 1$. This number p is larger than any of the primes, and it cannot itself be prime since it is distinct from all of the primes. So it must be divisible without remainder by a prime. But in fact dividing p by any prime leaves a remainder of 1. We have reached a contradiction! Therefore, our initial supposition must be false; there must be infinitely many prime numbers.

Practice (in seminar)

Let's diagram the mathematical *reductio* described above.

Homework (after seminar)

Think about the relationships between argument by *reductio*, *modus tollens*, and argument by elimination. (I will not collect this homework assignment.)

[1] I borrow this Euclid-inspired proof from http://www.math.utah.edu/~pa/math/q2.html.